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Agent-based simulation in market and production system

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Abstract: The Nash bargaining solution of oligopolies is introduced, and agent-based simulation model and procedure are developed, in which the firms are able to learn and approach the cooperative solution. An example with three firms illustrates the methodology, in which both sequential and simultaneous moves are allowed.

Keywords: oligopoly; bargaining solution; agent-based simulation.

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1 Introduction

Oligopoly model is a description of a special economic situation in which a particular market is controlled by a small group of firms. The number of firms is so small that the actions of any one of them will affect the price and have a measurable impact on the competitors. In USA, oligopolistic industries include accounting and audit services, tobacco, beer, aircraft, military equipment, motor vehicle, film and music recording industries among others.

Oligopoly theory is the most frequently discussed economic model class in the literature. The simplest model originates from the pioneering work of Cournot (1838). It models a realistic economic situation. Consider an industry with n firms that produce identical good or offers identical service. Let x_k ($k = 1, 2, \ldots, n$) denote the quantity offered by firm k, then its cost, $C_k(x_k)$, depends on its own production level, but the price, $f(s)$, depends on the total quantity $s = x_1 + \ldots + x_n$ offered to the market. The firms are assumed to have finite capacity limits, L_k, and their profits are the differences of their revenues and costs. The revenue of firm k is $x_k f(s)$ with cost $C_k(x_k)$, so the profit of this firm is

$$\varphi_k(x_1,\ldots,x_n) = x_k f\left(\sum_{i=1}^{n} x_i\right) - C_k(x_k). \tag{1}$$

Cournot considered this situation as an n-person game, where the firms are the players, the strategy set of player k is the finite, closed interval $[0, L_k]$, and its payoff function is given by equation (1). Following the work of Cournot, several static and dynamic extensions of this classical model have been developed.

In the early stages of the research, the existence and uniqueness of the equilibrium was the central issue (Nash, 1951). The main result can be given as follows:

Assume that the price function f and all cost functions C_k, $k = 1, \ldots, n$, are twice continuously differentiable, furthermore

i $f' < 0$

ii $x_k f'' + f' \le 0$

iii $f' - C'' < 0$

for all feasible x_k values. Then there is a unique equilibrium.

Note that condition i is a natural requirement, since any increase in the supply has to decrease the market price. Conditions ii and iii guarantee that φ_k is strictly concave in x_k, and they are clearly satisfied if f is concave and all cost functions C_k are convex.

A comprehensive summary of the earlier results is given in Okuguchi (1976) and their multi-product generalisations with some applications are presented in Okuguchi and Szidarovszky (1999).

There are several practical methods to find equilibrium, however the computation of cooperative solutions is a difficult task. The most common cooperative solution concept is the Nash bargaining solution which requires the optimisation of the product of the profit functions of all firms. This objective function is non-concave, so there is no efficient way to find its maximum. In this paper, we will introduce agent-based simulation when only a single-dimensional optimisation problem is solved at each step.

This concept is a realistic description of real economic situations, since the different firms – even if they cooperate – want to maximise their objective functions or the commonly agreed objective function by selecting their own best production plans. The sequence of optimal decisions can also be interpreted as an iteration procedure that can be used for the computation of the cooperative solution of the oligopoly game. This paper introduces an agent-based simulation model in which the firms are able to learn the Nash bargaining solution by a dynamic process that converges to this solution.

2 A linear model and equilibrium

In this paper, we study single-product oligopolies and assume that the price function f and the cost functions C_k are all linear:

$$f(s) = A - Bs, \tag{2}$$

$$C_k(x_k) = a_k + b_k x_k, \tag{3}$$

where $s = \sum_{i=1}^{n} x_i$ is the total industry output, B and A are positive constants, A is the maximum price and B is the marginal price, b_k is the marginal cost, and a_k is the fixed cost. In this case, all conditions of the general existence theorem are satisfied and therefore there exists a unique equilibrium. The linearity of the price and cost functions serves only mathematical convenience. More complex, nonlinear cases can be discussed in an analogous manner.

The profit of each agent k is represented as

$$\varphi_k = x_k\left(A - B\cdot\sum_{i=1}^{n} x_i\right) - (a_k + b_k x_k) = -B\cdot x_k^2 + (A - b_k)\cdot x_k - B\cdot x_{-k}\cdot x_k - a_k, \tag{4}$$

where $x_{-k} = \sum_{i \ne k} x_i$ is the total output of the rest of the industry.

From equation (4), without assuming cooperation among the agents, each agent's best response is obtained from the first order condition:

$$\frac{\partial \varphi_k}{\partial x_k} = -2Bx_k + A - b_k - Bx_{-k} = 0, \tag{5}$$

that is,

$$x_k = -\frac{1}{2}x_{-k} + \frac{A - b_k}{2B} = -s + \frac{A - b_k}{B}, \tag{6}$$

where $s = \sum_{i=1}^{n} x_i$ is the total supply, as before. Here, we assume that x_k is interior.

By adding these equations for all values of k, we have

$$s = -ns + \frac{nA - \sum_{k=1}^{n} b_k}{B}.$$

So

$$s = \frac{nA - \sum_{k=1}^{n} b_k}{(n+1)B}, \tag{7}$$

and from equation (6), the equilibrium production level of firm k is the following:

$$x_k = \frac{A + \sum_{l=1}^{n} b_l - (n+1)b_k}{(n+1)B}. \tag{8}$$

In the nonlinear case, the first order conditions provide n nonlinear equations for the n unknown production levels, x_k, and usually computer methods are used to find the solution. These methods are usually iterative and sufficiently good initial approximations are required to guarantee convergence.

3 Cooperative solutions

The equilibrium is the solution of the game, if it is considered non-cooperative. If there is certain level of cooperation between the firms, then cooperative game theoretical methods have to be applied. It is well known that by certain levels of cooperation the firms are able to increase their profits (see for example, Kopel and Szidarovszky, 2006). Most solution concepts are based on the characteristic function of the game. It is a real-valued function defined on all possible coalitions of the n players. Since there are 2^n possible subgroups (including the empty set), there are 2^n values of the characteristic function. By definition, $v(\phi) = 0$, and $v(\{1, \ldots, n\}) = \max \sum_{k=1}^{n} \varphi_k(x_1, \ldots, x_n)$. For any coalition $C = \{i_1, \ldots, i_r\} \subseteq \{1, 2, \ldots, n\}$,

$$v(C) = \max_{\substack{i \in C \\ x_i}} \min_{\substack{l \notin C \\ x_l}} \sum_{k \in C} \varphi_k(x_1, \cdots, x_n). \tag{9}$$

The value of the characteristic function shows the minimal obtainable total payoff of coalition C regardless of the actions of the other firms. The most common cooperative concepts can be found in the game theory literature (see for example, Forgo et al., 1999).

Another class of solutions is based on bargaining. There are several concepts and methods to find bargaining solutions. The most popular method is the Nash bargaining solution, which maximises the Nash-product

$$\prod_{k=1}^{n} (\varphi_k(x_1, \cdots, x_n) - \varphi_{k^*}), \tag{10}$$

where φ_{k^*} is a 'status quo' value, which might be the individual minimum value of the payoff of player k, it can also be the equilibrium payoff, or simply zero, since in the absence of agreement no firm will participate in the industry, so no profit is obtained. By selecting $\varphi_{k^*} = 0$ for all players, linear price and cost functions, the Nash bargaining solution is obtained by solving the following complicated optimization problem:

$$\text{Maximise} \prod_{k=1}^{n} \left(x_k \left(A - B\sum_{l=1}^{n} x_l \right) - (a_k + b_k x_x) \right)$$

$$\text{subject to } \quad 0 \le x_k \le L_k$$

$$\left.\begin{array}{l} 0 \le x_k \le L_k \\ x_k \left(A - B\sum_{l=1}^{n} x_l \right) - (a_k + b_k x_k) \ge 0 \end{array}\right\} (1 \le k \le n). \tag{11}$$

Since this is a nonconvex problem, there is no guarantee that any algorithm will give global optimum and does not terminate at a local optimum. Instead of applying a multidimensional optimisation routine, an agent-based simulation method can be suggested, in which the firms (players) are the agents, and at each time period each agent adjusts its own output to optimise the Nash product. So at each step only a single dimensional problem is solved. As it was also mentioned earlier, this process givens a realistic model of real economic systems.

4 The simulation study

We consider only the classical Cournot model, other model variants can be discussed similarly. We also assume that the firms know the true price and all cost functions and the past simultaneous output values of all firms. In this case firm k is able to find its best response $x_k^*(t)$ at any time t.

In the case of non-cooperative firms the best response is given by equation (6). In the case of cooperative firms, the best response of firm k is the output level that maximises the Nash product equation (10) with given output levels of the competitors. Note that the computation of the cooperative best responses requires the solution of a single-dimensional optimisation problem only. If $x_k(t)$ is the current output of the firm and $x_k^*(t)$ is its best response, then under discrete time scales the firm will change its output in the direction toward the desired output, since it cannot have large output 'jumps' instantaneously. Therefore the output change of the firms can be described by:

$$x_k(t+1) = x_k(t) + \alpha_k (x_k^*(t) - x_k(t)) \tag{12}$$

where α_k is the learning rate of firm k. However, at the next time period, the other firms may also adjust their outputs, so the market price might change, and therefore best response of firm k will not be optimal any more.

We used agent-based simulation to examine the long-term behaviour of the firms. We assumed cooperative agents with the Nash bargaining solution. For the sake of simplicity, we assumed only three firms and identical learning rates.

First, we consider the case of symmetric agents with identical cost function. Figure 1 shows the learning process of symmetric agents with learning rate $\alpha = 0.5$, where $A = 300$, $B = 1$, $a_1 = a_2 = a_3 = 10$, $b_1 = b_2 = b_3 = 1$ and the initial outputs of the three firms are $x_1 = 10$, $x_2 = 40$ and $x_3 = 120$, respectively. In this case with symmetric agents, the agents have the same costs and production abilities, therefore their final outputs are also the same after convergent learning regardless of the difference in the initial production values. Figure 2 shows the learning process with non-symmetric agents, where $a_1 = 100$,

$b_1 = 1$, $a_2 = 20$, $b_2 = 1$, $a_3 = 1000$, $b_3 = 1$, and the market price model is the same as in the previous case. In both Figures 1 and 2, firm 1's output is represented with dotted line, firm 2's output is represented with broken line, and firm 3's output with continuous line. In the case of non-symmetric firms, the firms have the same marginal cost but firm 3's fixed cost is much larger than those of firm 1 and firm 2. However, in Figure 2, firm 3's output is larger than those of the other firms. This learning result seems to be in conflict with intuition; however, it can be understood since the bargaining solution tries to reach global optimum of the Nash product which is the common objective of the firms, instead of looking for individual optimal solutions.

Figure 1 Learning process with symmetric agents (see online version for colours)

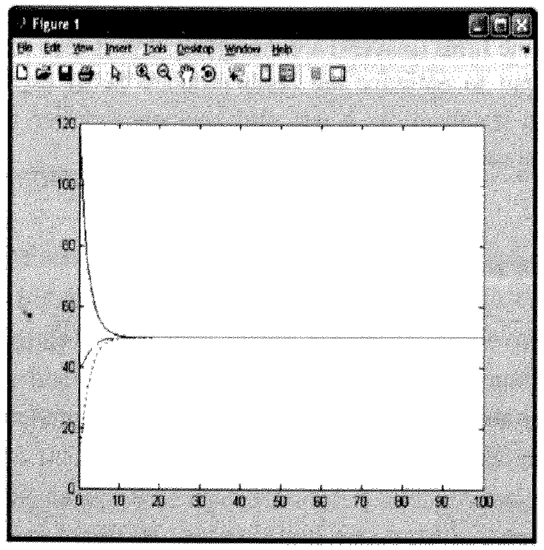
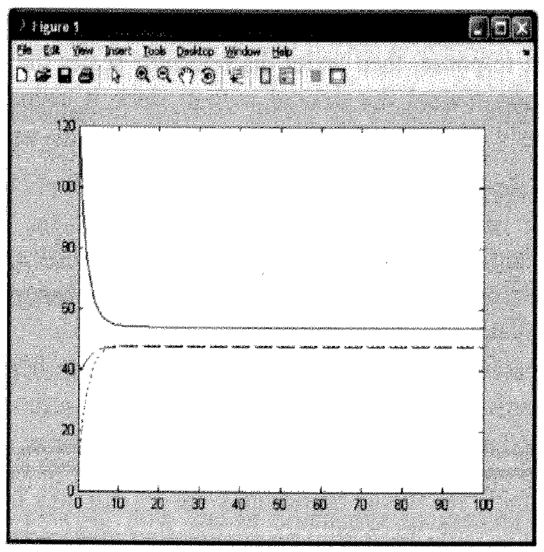


Figure 2 Learning process with non-symmetric agents (see online version for colours)



In order to further study our model we will examine the effect of different marginal costs. We study the case with non-symmetric agents, where the agents have the same fixed cost but different marginal costs. The common fixed cost is 1000 in the first graph, marginal costs are 1, 10, and 100, respectively, in the first graph; the common fixed cost is 1000 and marginal costs are 1, 111, and 333, respectively, in the second graph of Figure 3. The marginal costs of firm 1, firm 2 and firm 3 are selected in ascending order. However, the sequences of the limiting output values are different and it shows that there is no association between the limiting output values and the marginal cost values.

Figure 3 The learning processes of agents with various marginal costs (see online version for colours)

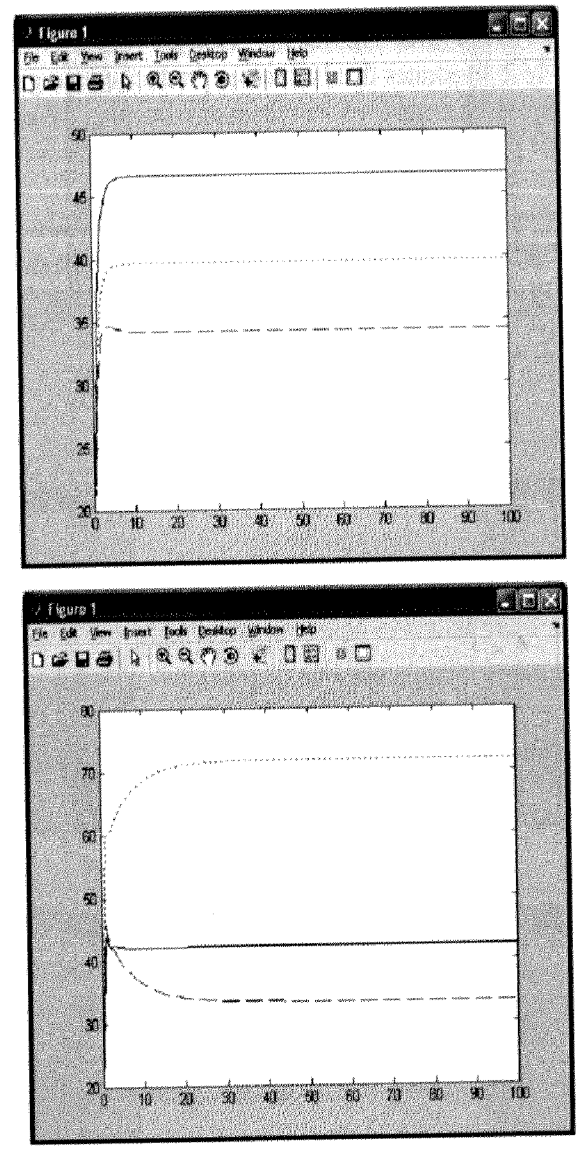


Figure 4 shows the learning process with various learning rates $\alpha = 0.2$, 0.5, 1.0 and 1.4, respectively. The figure shows that the speed of convergence increases with higher learning rate as $\alpha \leq 1$. When α is larger than one, learning does not converge any more. This observation is consistent with learning in non-cooperative oligopoly games. Okuguchi and Szidarovszky (1999) have proved that in equilibrium based learning the equilibrium is locally asymptotically stable if

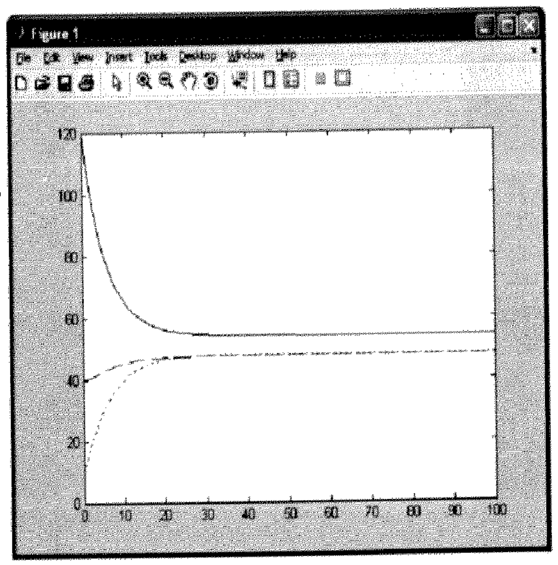
$$\alpha_k < 4 \quad \text{and} \quad \sum_{k=1}^{n} \frac{\alpha_k}{4-\alpha_k} < 1, \tag{13}$$

where α_k is the learning rate for agent k. Assume as a special case that the agents are symmetric with identical learning rate, then the convergence condition becomes

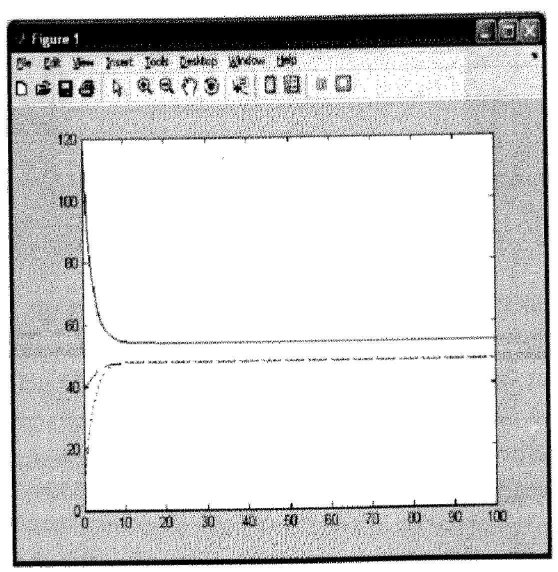
$$\alpha < \frac{4}{n+1}. \tag{14}$$

We also present a comparison of simultaneous and sequential moves of the firms with three firms as before. We have selected $A = 300$, $B = 1$, $a_1 = a_2 = a_3 = 10$, $b_1 = b_2 = b_3 = 1$ and the initial outputs of the three firms as $x_1 = 10$, $x_2 = 40$ and $x_3 = 120$. The results are shown in Figure 5 which illustrates that both cases produce convergent strategies and the convergence is faster in the case of simultaneous learning. We have repeated this experiment with other parameters, and always reached the same conclusion.

Figure 4 Learning processes with various learning rates: (a) $\alpha = 0.2$; (b) $\alpha = 0.5$; (c) $\alpha = 1.0$ and (d) $\alpha = 1.4$ (see online version for colours)

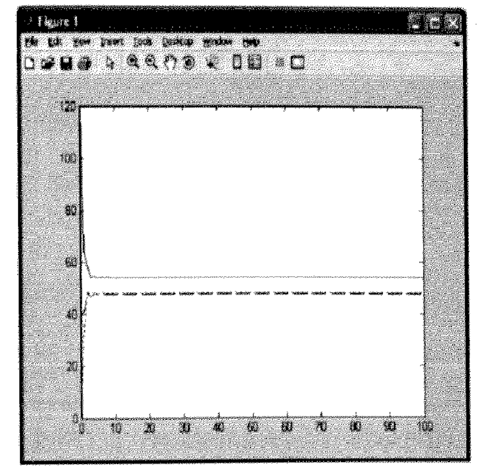


(a)

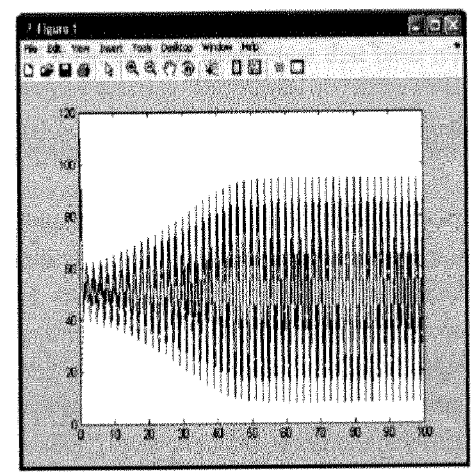


(b)

Figure 4 Learning processes with various learning rates: (a) $\alpha = 0.2$; (b) $\alpha = 0.5$; (c) $\alpha = 1.0$ and (d) $\alpha = 1.4$ (see online version for colours) (continued)

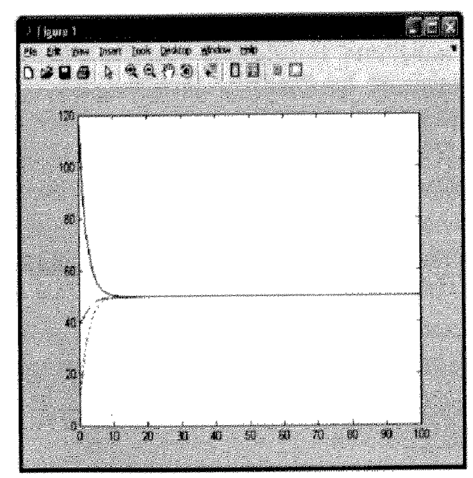


(c)



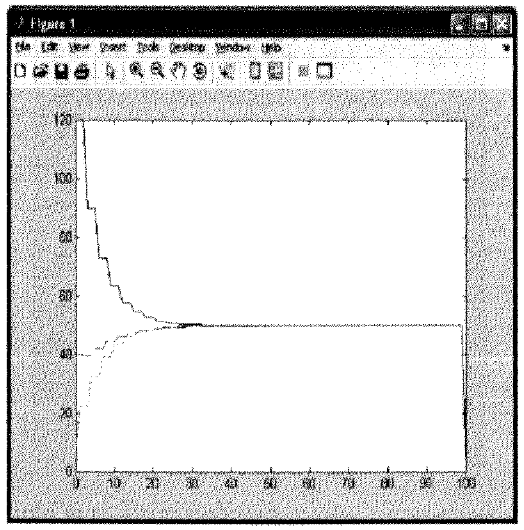
(d)

Figure 5 Comparison of simultaneous and sequential learning: (a) simultaneous learning and (b) sequential learning (see online version for colours)



(a)

Figure 5 Comparison of simultaneous and sequential learning: (a) simultaneous learning and (b) sequential learning (see online version for colours) (continued)



(b)

5 Conclusions

In this paper, oligopoly models were examined under the assumption that the firms seek Nash bargaining solution by cooperation in order to increase their profits. We illustrated with agent-based simulation that the firms are able to learn and reach the bargaining solution by both simultaneous and sequential dynamic learning processes. We assume that at each step each agent finds its best response with given output levels of the competitors and moves into its direction. We have examined symmetric and asymmetric firms as well as the effect of differences in fixed, any marginal costs and learning rates.

Our study was based on the Nash bargaining solution, the case of other bargaining and cooperative solutions can be examined in a similar way, which will be the topic of our future paper.

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Agent-based simulation of a simple market

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Abstract: We present an agent-based iterated simulation of a simple market structure. The simulation is based on trade and price satisfaction (payoff) functions for both buyers and sellers. The participants are trying to follow the behaviour of those of their neighbours that have received the highest payoff in the previous iteration. This results in trading and transformation of buyers into sellers and vice versa. Equilibrium conditions develop. The ratio of buyers and sellers fluctuates around this equilibrium. We have investigated two cases: slight and strong domination of the price satisfaction over the trade satisfaction. The results show how the global equilibrium condition depends on the local price. The simulation is a special case of the Chicken Game.

Keywords: agent-based simulation; Chicken Game; market simulation.

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