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Modelling of Eventistic Systems

A special class of eventistic systems is considered in the paper. Problems of their modelling, decomposition and synthesis are presented. Theorems concerning direct and indirect computer simulation of decomposed eventistic systems have been given.

Introduction

The considerations of this paper concern a special class of systems called the eventistic systems.

We use a notion of event for the purpose of creating a formal model of a phenomenon and establishing relationships among the objects of that phenomenon. To know how the system behaves, acts, say, "works" means to analyse its formal model first. Since mathematical models of systems are usually very complicated they are often examined with a help of a computer. Here the question arises whether eventistic systems might be analysed in the way of computer simulation.

It seems that (because of the way they are described in) they are very suitable to simulation, particularly if one uses the event-oriented programming languages.

Our objective in this paper is to consider the synthesis problem of an eventistic system's model suitable to computer simulation. Such a model could be created in the direct way. However, there are many cases (mainly caused by the formal models complexity) where decomposition should be applied first and only then a model of the decomposed eventistic system may be synthesized.

Because of the above mentioned reasons we would like to present problems of eventistic systems decomposition as well as considerations on direct synthesis of a model of simulation, and indirect one based on the decomposed system.

1. Elementary Eventistic Systems

In this chapter we shall introduce a special type of an eventistic system presented in [3–5, 12].

The reason why we deal with systems of this type is that they are suitable to computer aided simulation which is, among other things, an object of our considerations. So, let us present the concept of an elementary eventistic system.

DEFINITION 1. An elementary eventistic system ε is a binary relation on Carthesian product of the set of events X

$$\varepsilon \subset X \times X, \tag{1}$$

where: $X \subset T \times C$,
T — the time-set,
C — the state-set.

It is easy to prove that the set of events is a subset of the so-called generic set $\mathfrak{B}$. Therefore the system ε is a particular kind of a general eventistic system [5, 7].

In further considerations we treat systems as complete elementary. Thus holds the equation

$$D\varepsilon = X \tag{2}$$

($D\varepsilon$ denotes the domain of ε)

Theorem 1. Every eventistic system has a functional parametric representation

$$\hat{\varepsilon}: X \times \Omega \to X, \tag{3}$$

where Ω is the set of parameters,
and the consistency postulate holds i.e.

$$\langle x, y \rangle \in \varepsilon \Leftrightarrow (\exists \omega)(\hat{\varepsilon}(x, \omega) = y). \tag{4}$$

Proof: This is proven in [9].

Since a functional description of a system exists we restrict our considerations to the functional eventistic system

$$\varepsilon: X \to X$$

(according to Theorem 1 a system ε given as a relation may be represented by the function $\hat{\varepsilon}$).

Such systems are an object of the considerations on possibilities of their modelling. We are also interested in a problem of investigating their properties with the help of a computer. Computer modelling of eventistic systems is preceded by problems of decomposition, particularly useful in a process of model's synthesis. Therefore we introduce a notion of a modelling relation and a definition of a hierarchical connection of elementary eventistic systems.

DEFINITION 2. Let two systems $\varepsilon_1: X_1 \to X_1$ and $\varepsilon_2: X_2 \to X_2$ be given. The eventistic system ε_2 modells the system $\varepsilon_1 (\varepsilon_2 \sim \varepsilon_1)$ if there exists an event coding function φ

$$\varphi: X_1 \to X_2 \tag{5}$$

such that the following diagram

$$\begin{array}{ccc} X_1 & \xrightarrow{\varepsilon_1} & X_1 \\ \varphi\downarrow & & \downarrow\varphi \\ X_2 & \xrightarrow{\varepsilon_2} & X_2 \end{array}$$

is commutative.

The relation of modelling in our sense is adequate to the general systems modelling relation introduced in [8, 10, 14].

COROLLARY 1. If the eventistic system ε_2 modells the system ε_1 then

$$\langle X_1, \varepsilon_1\rangle \xrightarrow{\text{HOM}} \langle X_2, \varepsilon_2\rangle,$$

where HOM denotes a homomorphism,
if the coding function is bijective then

$$\langle X_1, \varepsilon_1\rangle \xrightarrow{\text{ISM}} \langle X_2, \varepsilon_2\rangle$$

ISM denotes an isomorphism.
Hence, we consider ε_2 as the isomorphic model of ε_1
Proof: The result comes from definition 2.

2. Hierarchical Decomposition of Eventistic Systems

Our objective in this chapter is to determine a hierarchical connection of elementary eventistic systems and, then, to introduce a definition of their hierarchical decomposition. Assume that k elementary systems are given $(k \in K \subset N)$

$$(\forall i \in K)(\varepsilon_i : X_i \rightarrow X_i). \tag{6}$$

In order to define their connection it is necessary to know their co-operation represented by the co-operation functions ξ_{ij} determined as follows

$$(\forall i, j \in K)(\xi_{ij} : X_i \times X_j \rightarrow X_j). \tag{7}$$

As to satisfy the consistency condition [5] the equation

$$(\forall i \in K)\big(\xi_{ii}(\varepsilon_i(x_i), x_i) = \varepsilon_i(x_i)\big) \tag{8}$$

must hold.

On the basis of the present event of the system ε_j and the action of the system ε_i the co-operation function ξ_{ij} determines a new event of the system ε_j. This new event has to be simultaneous with the event of ε_i i.e.

$$\text{crd}^1 \xi_{ij}(x_i, x_j) = \text{crd}^1 x_i. \tag{9}$$

For the purpose of synchronizing the actions of the systems $\{\varepsilon_i | i \in K\}$ we introduce a co-ordinating function μ

$$\mu : Z \to K, \tag{10}$$

where $Z \subset X\{X_i | i \in K\}$ is the set of ordered sequences of simultaneous events.

DEFINITION 3. Let systems $\{\varepsilon_i | i \in K\}$, the co-operation functions ξ_{ij} and the co-ordinating function μ (determined by (6), (7), (10) be given.

A hierarchical connection of systems $\{\varepsilon_i | i \in K\}$ is a system

$$F : Z \to Z \ (F = \underset{K}{|+|} \varepsilon_i) \tag{11}$$

such that

$$F(z) = \langle \xi_{\mu(z),j}(\varepsilon_{\mu(z)}(\mathrm{crd}^{\mu(z)} z),\ \mathrm{crd}^j z) | j \in K \rangle. \tag{12}$$

(The fact that $\underset{K}{|+|}\varepsilon_i$ is also an eventistic system is shown in [5]). The hierarchical connection is illustrated in Figure 1.

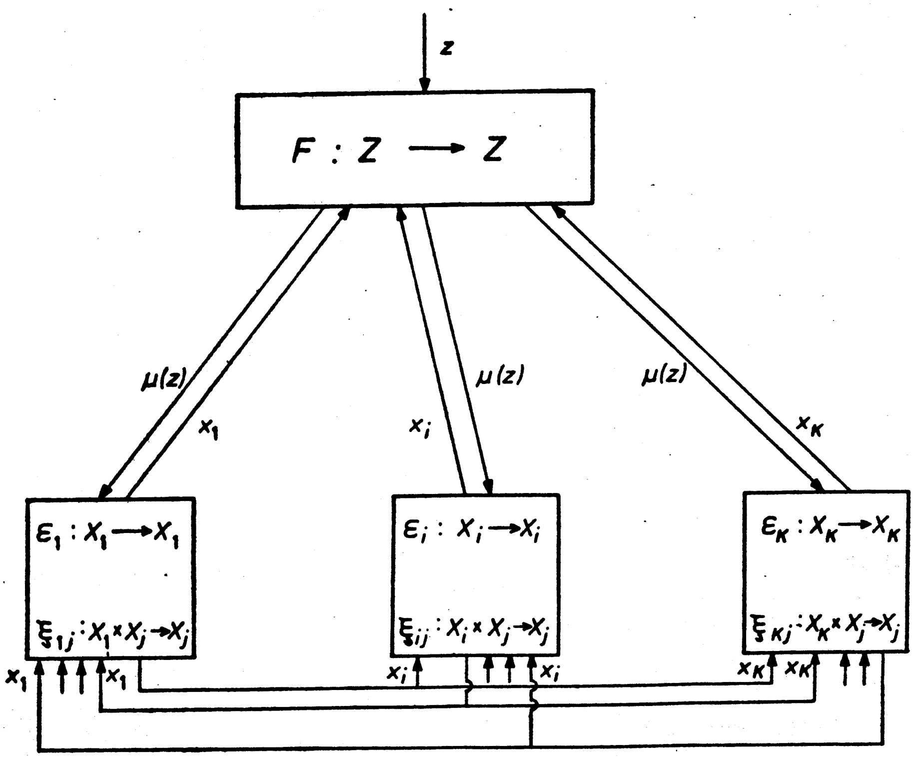

Fig. 1

DEFINITION 4. The elementary eventistic systems $\{\varepsilon_i | i \in K\}$ are a hierarchical decomposition of an elementary system $\varepsilon \subset X \times X$ if their hierarchical connection modells the system ε

$$|+|\varepsilon_i \sim \varepsilon .$$

The question, then, is whether each elementary eventistic system has a hierarchical decomposition. We have provided the answer to this question in the following theorem.

Theorem 2. An elementary eventistic system $\varepsilon: X \to X$ has an isomorphic hierarchical decomposition if the conditions

(i) $X \subset T \times C$ and $C = X\{C_i | i \in K \subset N\}$,

(ii) $\{i_x | x \in X\} = K$ where $i_x = \min\{i \in K | \mathrm{crd}^i \mathrm{crd}^2 x \neq \mathrm{crd}^i \mathrm{crd}^2 \varepsilon(x)\}$,

(iii) $(\forall x, x')(i_x = i_x \wedge t_x = t_{x'} \Rightarrow (t_{\varepsilon(x)} = t_{\varepsilon(x')}) \wedge$
$\wedge ((\forall j \in K)(\mathrm{crd}^j \mathrm{crd}^2 \varepsilon(x) \neq \mathrm{crd}^j \mathrm{crd}^2 \varepsilon(x') \Rightarrow \mathrm{crd}^j \mathrm{crd}^2 x \neq \mathrm{crd}^j \mathrm{crd}^2 x')))$

where $t_x = \mathrm{crd}^1 x$

hold.

Thus the following diagram

$$\begin{tikzcd} X \arrow[r, "\varepsilon"] \arrow[d, "\varphi"'] & X \\ Z \arrow[r, "\underset{K}{|+|}\varepsilon_i"] & Z \arrow[u, "\varphi^{-1}"'] \end{tikzcd}$$

is commutative.

Proof: The theorem is proven in [5]. The precise forms of the functions ξ_{il} and μ are given there too. The hierarchical decomposition is directly applicable to synthesis of the so-called eventistic switching circuits [2].

In the next chapter we shall present problems connected with simulation of eventistic systems.

3. Computer Simulation of Eventistic Systems

3.1. Direct Simulation

We assume the computer modelling to be a process of searching for an appriopriate (to a given system) algorithm of simulation. Such an algorithm is, in the sense of Pawlak [11], a pair

$$A = \langle V, f \rangle \tag{13}$$

where V is termed the set of computer's events (states) and f is the iterative events transition function. While a program's statement runs a computer's event represents the state of all registers. So, if one uses the event-oriented programming languages of simulation, the event transition function may be represented by a sequence of program's statements [1, 9, 13]. This leads to the question: do there exist possibilities of simulating eventistic systems with the aid of a computer? If so, what conditions must hold then?

The following theorem provides the answer to these questions.

Theorem 3. An elementary eventistic system $\varepsilon: X \to X$ is simulatable if there exists an event coding function η such that

$$\eta: X \to V \ (\eta \text{ is bijective}). \tag{14}$$

It is easy to notice that the system ε is, then, isomorphic to an algorithm of simulation i.e.

$$f(v) = \eta\big(\varepsilon(\eta^{-1}(v))\big). \tag{15}$$

Since there could exist some troubles with determination of the events coding function, particularly in case of events with the large dimension of the state vector, the simulation of a system might not be possible. Representation of the events transition function may be difficult as well. If it is not possible to simulate a system because of the above reasons, the synthesis of a systems' model with the help of the hierarchical decomposition (if only such exists) is very useful. We shall present this problem in the following part.

3.2. Indirect Simulation of Eventistic Systems

LEMMA 1. A hierarchical connection $|\underset{k}{+}|\ \varepsilon_i$ is simulatable if the component systems ε_i are simulatable too.

Proof. In order to prove this lemma we shall try to build up an algorithm of simulation isomorphic to $|\underset{k}{+}|\ \varepsilon_i$

According to the assumption

$$(\forall i \in K)(\eta_i: X_i \to V_i)(\eta_i \text{ is bijective}). \tag{16}$$

We set up an algorithm

$$\bar{A} = \langle \bar{V}, \bar{f} \rangle, \tag{17}$$

where

$$\bar{V} \subset X\{V_i | i \in K\},$$

and

$$\bar{f}: \bar{V} \to \bar{V}$$

with

$$\bar{f}(\langle v_1, \ldots, v_k \rangle) = \psi\mu(\eta_1^{-1}(v_1), \ldots, \eta_k^{-1}(v_k))(\langle v_1, \ldots, v_k \rangle), \tag{18}$$

where

$$(\forall i \in K)(\psi_i: \bar{V} \to \bar{V}) \tag{19}$$

and

$$\psi_i(\langle v_1, \ldots, v_k\rangle) = \langle \eta_j\big(\xi_{ij}\big(\eta_i^{-1}\big(f_i(v_i)\big), \eta_i^{-1}(v_j)\big)\big) | j \in K\rangle. \tag{20}$$

Introduce

$$\bar{\eta}: Z \to \bar{V} \tag{21}$$

determined as follows

$$\bar{\eta}(z) = \langle \eta_i(x_i) | i \in K\rangle \tag{22}$$

by (18), (20), (22) and definition 3.

$$\bar{f}(\bar{v}) = \psi^{-1}_{\mu(\eta(\bar{v}))}(\bar{v}) =$$

$$= \langle \eta_j\big(\xi_{\mu(\bar{\eta}^{-1}(\bar{v}))}\big(\varepsilon_{\mu(\bar{\eta}^{1}(\bar{v}))}(\mathrm{crd}^{\mu(\bar{\eta})^{-1}(\bar{v})})\big(\eta^{1-}(\bar{v})\big)_{(\bar{\eta}^{-1}(\bar{v}))}, \mathrm{crd}^j\bar{\eta}^{-1}(\bar{v})\big) | j \in K\rangle =$$

$$= \bar{\eta}\big(F(\bar{\eta}^{-1}(\bar{v})\big)$$

what results in $\langle Z, F\rangle \xleftrightarrow{\mathrm{ISM}} \langle \bar{V}, \bar{f}\rangle$

This proves the lemma.

We now use the proven lemma to synthesize a simulation algorithm of an elementary eventistic system. Such an algorithm simulates a given system by simulating its component systems and their co-operation.

Theorem 4. A system $\varepsilon: X \to X$ is indirectly simulatable if it satisfies the conditions of Theorem 3 and the components of its hierarchical decomposition are simulatable.

Proof. The result follows immediately from Theorem 3 and Lemma 1.

Thus the diagram

$$\begin{tikzcd}
X \arrow[r, "\varepsilon"] \arrow[d, "\varphi"'] & X \\
Z \arrow[r, "F"] \arrow[d, "\bar{\eta}"'] & Z \arrow[u, "\varphi^{-1}"'] \\
\bar{V} \arrow[r, "\bar{f}"] & \bar{V} \arrow[u, "\bar{\eta}^{-1}"']
\end{tikzcd}$$

is commutative.

4. Final Remarks

The results obtained hitherto lead to the following conclusions.

If only the system ε is hierarchically decomposable it is enough to build models of its component systems and co-ordinate their actions by the functions ξ_{ij} and μ.

Compare the function $\bar{\eta} = \langle \eta_1, \eta_2, \ldots, \eta_k \rangle$ with its components η_i. One may notice that the dimension of the events state vector of $\bar{\eta}$ is much bigger than the dimension of the state vector of events which are the arguments of η_i. This might make the coding (i.e. representing the systems events by the computer's ones) easier, what in fact leads to a simpler model of simulation.

The most appropriate to the considered class of problems are the event-oriented simulation languages as CSL and GPSS. Their structure apparently implies the simulation of the decomposed eventistic systems.

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Modelowanie systemów zdarzeniowych

Streszczenie

W pracy rozważana jest specjalna klasa systemów zdarzeniowych. Omówiono problemy modelowania, dekompozycji i syntezy systemów zdarzeniowych. Przedstawiono twierdzenia dotyczące pośredniej i bezpośredniej symulacji komputerowej zdekomponowanych systemów zdarzeniowych.

Моделирование эвентистических системов

Резюме

В статьи представлено специальный класс эвентистических системов, которые можна програмировать языками событий на вычислительной машине. Представлено тоже их мо-

делирования, декомпозиции и синтезы. В статьи содержатся теоремы о беспосредним и посредним вычислительным модели рованию с помощью декомпозиции.

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